ADDENDUM TO
“NEW CHARACTERIZATIONS OF BERGMAN SPACES”

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Two problems have been brought to our attention since the publication of our paper “New characterizations of Bergman spaces” [5] which will be referred to as “the paper” in what follows. The first issue concerns a result which we thought was well-known but was actually not quite so. The second issue concerns a lack of details in a major step of the proof of Theorem 2 in the paper. We will clarify these issues in this addendum.

In addition, Kwon sent [3] to the first named author before our paper was accepted for publication, but we failed to acknowledge Kwon’s paper. We wish to apologize here. Kwon’s paper [3] proves the one-dimensional case of our main results for Bergman spaces with more general weights under the additional assumption that \( f(0) = 0 \). The one dimensional case of the Littlewood–Paley inequality can also be found in [1]. Related work for Hardy spaces on the unit ball can be found in [6].

We asserted in the paper that Lemma 9 could be found in [4]. This is not the case. The first inequality in Lemma 9,

\[
\int_{B_n} |f|^p \, dv \leq C \left[ |f(0)|^p + \int_{B_n} |f(z)|^{p-2} |\overline{\nabla} f(z)|^2 \, dv(z) \right],
\]

was not used anywhere in the paper. However, the second inequality in Lemma 9,

\[
|f(0)|^p + \int_{B_n} |f(z)|^{p-2} |\overline{\nabla} f(z)|^2 \, dv(z) \leq C \int_{B_n} |f|^p \, dv,
\]

was used in the paper to prove the inequality

\[
|f(0)|^p + I_4(f) \leq CI_1(f).
\]

To prove (2), we use the identity

\[
\int_{B_n} |f|^p \, dv = |f(0)|^p + c_{p,n} \int_{B_n} |\overline{\nabla} f(z)|^2 |f(z)|^{p-2} G_1(z)(1 - |z|^2)^{-n-1} \, dv(z),
\]

where

\[
G_1(z) = \int_{|z|}^1 \frac{(1 - t^2)^{n-1}(1 - t^{2n})}{t^{2n-1}} \, dt.
\]
Identity (3), which was stated as Exercise 4.5 in [7], follows from Theorem 4.23 of [7] and integration in polar coordinates. Since
\[ G_1(z) \geq \int_{|z|}^{1} \frac{(1-t)^{n-1}2nt^{2n-1}(1-t)dt}{t^{2n-1}} = \frac{2n}{n+1} (1-|z|)^{n+1}, \]
we obtain inequality (2).

Taking \( q = 2 \) in Theorem 2 of the paper, we obtain Lemma 9 as a consequence.

There is a second point in the paper that warrants clarification. Namely, we proved that for \( p < q < p + 2 \),
\[ cI_1(f) \leq |f(0)|^p + I_2(f)^{\frac{1}{p}}I_1(f)^{\frac{1}{q}}, \]
and we then wrote that from this one easily deduces
\[ I_1(f) \leq C[|f(0)|^p + I_2(f)]. \]
This is true if \( f \) is holomorphic in a neighborhood of the closed ball. If \( f \) is arbitrary, we want to apply this fact to the functions
\[ f_\rho(z) = f(\rho z), \quad 0 < \rho < 1, \]
to get
\[ \frac{1}{\rho^{2n+2\alpha}} \int_{\rho B_n} |f(w)|^p (\rho^2 - |w|^2)^\alpha \, dv(w) \leq C|f(0)|^p + \frac{C}{\rho^{2n+2\alpha+2q}} \int_{\rho B_n} |Rf(w)|^q |f(w)|^{p-q}(\rho^2 - |w|^2)^{q+\alpha} \, dv(w). \]
When \( q + \alpha \geq 0 \), we let \( \rho \to 1^- \) and apply Fatou’s lemma on the left hand side and the monotone convergence theorem on the right to get the desired result. However, if \( q + \alpha < 0 \), which implies \( p < 1 \), we cannot apply the monotone convergence theorem. In this case, we can use a trick due to Kwon [2, 3] as follows. From the Littlewood–Paley type inequality
\[ M_p^p(r, f) \leq C|f(0)|^p + C \int_{\rho B_n} (\rho - |z|)^{p-1} |Rf(z)|^p \, dv(z), \quad p < 1, \quad \frac{1}{2} < \rho < 1, \]
we get as in [2, 3]
\[ \int_{\rho B_n} |f(z)|^p(1-|z|^2)\alpha \, dv(z) \leq C|f(0)|^p + C \int_{\rho B_n} |Rf(z)|^p(1-|z|^2)^{p+\alpha} \, dv(z). \]
If we replace \( I_1(f) \) and \( I_2(f) \) by \( I_1(\rho, f) \) and \( I_2(\rho, f) \), respectively, where
\[ I_1(\rho, f) = \int_{\rho B_n} |f(z)|^p \, dv_\alpha(z), \]
and
\[ I_2(\rho, f) = \int_{\rho B_n} |f(z)|^{p-q}(1-|z|^2)Rf(z)^q \, dv_\alpha(z), \]
we obtain
\[ I_1(\rho, f) \leq C[|f(0)|^p + I_2(\rho, f)]. \]
Let \( \rho \to 1 \) then we obtain the desired result.
References


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