CORRIGENDUM TO “ATOMIC DECOMPOSITION OF HARDY–MORREY SPACES WITH VARIABLE EXPONENTS”

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Abstract. We correct a technical error in [2, Theorem 5.1].

In this note, we correct a technical error appeared in the proof of [2, Theorem 5.1]. At the end of p. 49 of [2], we use the inequalities
\[ c|\varphi \ast g^j(x)| \leq (\mathcal{M}g^j)(x) \leq (\mathcal{M}f)(x) \chi_{\{x \in \mathbb{R}^n: (\mathcal{M}f)(x) \leq 2^j\}}(x) + 2^j \sum_{k \in \mathbb{N}} \frac{l(Q_j^k)n+d+1}{(l(Q_j^k) + |x - x_j^k|)^{n+d+1}} \]
\[ \leq C2^j \]
to prove that \( g^j \to 0 \) in \( S'(\mathbb{R}^n) \) as \( j \to -\infty \). This is an error as the last inequality does not necessarily hold.

Most importantly, the result \( \lim_{j \to -\infty} g^j = 0 \) in \( S'(\mathbb{R}^n) \) is valid. We now give a proof of the result \( \lim_{j \to -\infty} g^j = 0 \) in \( S'(\mathbb{R}^n) \) by using the ideas in [2, p. 50]. The reader is referred to [2] for the notions used in this note.

For any \( Q \in \mathcal{B} \), [2, Proposition 5.4] yields
\[ \int_Q |(\mathcal{M}g^j)(x)| \, dx \leq C2^j \int_Q dx + C2^j \int_Q \sum_{k \in \mathbb{N}} \frac{l(Q_j^k)n+d_p(\cdot)+1}{(l(Q_j^k) + |x - x_j^k|)^{n+d_p(\cdot)+1}} \, dx \]
\[ \leq C2^j |Q| + C2^j \sum_{k \in \mathbb{N}} \int_{\mathbb{R}^n} \chi_Q(x)((\mathcal{M}\chi_{Q_j^k})(x))^{(n+d_p(\cdot)+1)/n} \, dx. \]

By using [1, Chapter II, Theorem 2.12], we obtain
\[ \int_{\mathbb{R}^n} ((\mathcal{M}\chi_{Q_j^k})(x))^{(n+d_p(\cdot)+1)/n} \chi_Q(x) \, dx \leq \int_{\mathbb{R}^n} (\chi_{Q_j^k}(x))^{(n+d_p(\cdot)+1)/n}(\mathcal{M}\chi_Q)(x) \, dx \]
\[ = \int_{\mathbb{R}^n} \chi_{Q_j^k}(x)(\mathcal{M}\chi_Q)(x) \, dx \]
\[ = \int_{Q_j^k} (\mathcal{M}\chi_Q)(x) \, dx \]
because \( (n + d_p(\cdot) + 1)/n > 1 \).

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Therefore, the finite intersection property of \( \{Q_k^j\} \) yields
\[
\int_Q |(Mg^j)(x)| \, dx \leq C 2^j|Q| + C 2^j \sum_{k \in \mathbb{N}} \int_{Q_k^j} (M \chi_Q)(x) \, dx \\
\leq C 2^j|Q| + C 2^j \int_{O_j} (M \chi_Q)(x) \, dx.
\]

Consequently, for any \( \varphi \in \mathcal{S}(\mathbb{R}^n) \) and \( x \in \mathbb{R}^n \), we have
\[
|g^j * \varphi(x)| \leq C \frac{1}{|B(x, 1)|} \int_{B(x, 1)} |M_1^j(g^j, \varphi)(y)| \, dy \\
\leq C \int_{B(x, 1)} |(Mg^j)(y)| \, dy \\
\leq C 2^j|B(x, 1)| + C 2^j \int_{O_j} (M \chi_B(x, 1))(y) \, dx
\]
for some \( C > 0 \).

Thus, it suffices to show that \( 2^j \int_{O_j} (M \chi_B(x, 1))(y) \, dy \to 0 \) as \( j \to -\infty \).

Let \( 0 < r < \min(1, m_p(\cdot)) \) and \( B^k = B(x, 2^k) \setminus B(x, 2^{k-1}) \) when \( k \geq 1 \) and \( B^0 = B(x, 1) \). We find that
\[
\int_{O_j} (M \chi_B(x, 1))(y) \, dy \leq \int_{O_j} (1 + |x - y|)^{-n} \, dy \\
\leq C \sum_{k=0}^{\infty} 2^{-kn} \int_{O_j} \chi_B^k(y) \, dy \\
\leq C \sum_{k=0}^{\infty} \frac{1}{|B(x, 2^k)|} \|\chi_{O_j \cap B(x, 2^k)}\|_{L^p(\mathbb{R}^n)} \|\chi_{B(x, 2^k)}\|_{L^{p/(r')}_{\mathbb{R}^n}} \\
\leq C \sum_{k=0}^{\infty} \frac{u(x, 2^k)^r}{\|\chi_{B(x, 2^k)}\|_{L^{p/(r')}_{\mathbb{R}^n}}} \|\chi_{O_j}\|_{\mathcal{M}_p(\cdot, u)}.
\]

As \( u^r \in W_{\mathcal{H}_{p(\cdot)/r}} \), [2, Lemma 3.3] yields
\[
2^j \int_{O_j} (M \chi_Q(x, 1))(y) \, dy \leq 2^j C \|\chi_{O_j}\|_{\mathcal{M}_p(\cdot, u)}
\]
for some \( C > 0 \) independent of \( j \in \mathbb{Z} \).

In view of \( 0 < r < 1 \) and \( \|\chi_{O_j}\|_{\mathcal{M}_p(\cdot, u)} \leq 2^{-jr} \|\mathcal{M}f\|_{\mathcal{M}_p(\cdot, u)} = 2^{-jr} \|f\|_{\mathcal{H}_{p(\cdot), u}} \), we have
\[
\lim_{j \to -\infty} 2^j \int_{O_j} (M \chi_Q(x, 1))(y) \, dx \leq C \lim_{j \to -\infty} 2^{j-r} \|f\|_{\mathcal{H}_{p(\cdot), u}} = 0.
\]

References


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