ERRATUM TO “QUASICONFORMAL HARMONIC MAPPINGS WITH THE CONVEX HOLOMORPHIC PART”

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Corollary 2.7. Given $R > 0$ let $H$ be a conformal mapping in $D$ such that $H(D)$ is a convex domain and $D(H(0), R) \subset H(D)$. Then $H$ is co-Lipschitz with

\[ L^-(H) = D^-(H) \geq \frac{R}{4}. \tag{2.19} \]

Proof. Under the assumption of the corollary we see that the mapping $D \ni z \mapsto \tilde{H}(z) := H(z) - H(0)$ maps $D$ onto a convex domain and $D(0, R) \subset \tilde{H}(D)$. Since $\tilde{H}(0) = 0$ we conclude from [10, Corollary 3.1] (see also [5, Theorem 2.5]) that

\[ |H'(z)| = |\tilde{H}'(z)| \geq \frac{R}{4}, \quad z \in D. \]

Therefore $D^-(H) \geq R/4$, and so the inequality in (2.19) holds. By Remark 2.6 we see that the equality in (2.19) holds. Therefore $H$ is a co-Lipschitz mapping, which is our claim. \qed

Let us consider the following deformations of a harmonic mapping $F = H + G$ in $D$,

\[ D \ni z \mapsto F_\varepsilon(z) := H(z) + \varepsilon G(z), \quad \varepsilon \in \mathbb{C}. \tag{2.20} \]

Using now the decomposition (2.2) we derive the following theorem.

Theorem 2.8. Let $F = H + G$ be a sense-preserving harmonic mapping in $D$. Suppose that $H$ is injective, $H(D)$ is a rectifiably $M$-arcwise connected domain with a given $M \geq 1$ and that $F$ is not a conformal mapping. Then for every $\varepsilon \in D(1/M\|\mu_F\|_\infty)$, $F_\varepsilon$ is a quasiconformal harmonic mapping. Moreover, if $M = 1$, then $F_\varepsilon$ is co-Lipschitz.

Proof. Fix $\varepsilon \in D(1/M\|\mu_F\|_\infty)$. By setting $H(D) \ni z \mapsto \phi(z) := \varepsilon G \circ H^{-1}(z)$, we see that for every $z \in H(D)$,

\[ |\phi'(z)| = \left| \frac{\varepsilon G'(H^{-1}(z))}{H'(H^{-1}(z))} \right| = |\varepsilon| \|\mu_F(H^{-1}(z))\| \leq |\varepsilon| \|\mu_F\|_\infty. \tag{2.21} \]

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Hence $MD^+(\phi) \leq M|\varepsilon|\|\mu_F\|_\infty < 1$. From Lemma 2.4 it follows that $I[\phi]$ is bi-Lipschitz, and so $I[\phi]$ is quasiconformal. Since $F_\varepsilon = I[\phi] \circ H$, $F_\varepsilon$ is a quasiconformal mapping as a composition of quasiconformal ones. Suppose now that $M = 1$, i.e., $H(D)$ is a convex domain. By the conformality of $H$, $D(H(0), R) \subset H(D)$ for a certain positive number $R$. Then by Corollary 2.7 we see that $H$ is a co-Lipschitz mapping. Therefore $F_\varepsilon$ is a co-Lipschitz mapping as a composition of co-Lipschitz ones, which proves the theorem.

References


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