Annales Academiæ Scientiarum Fennicæ
Mathematica
Volumen 45, 2020, 305–312

LOCAL L^p-SOLUTION FOR SEMILINEAR HEAT EQUATION WITH FRACTIONAL NOISE

Jorge Clarke and Christian Olivera

Université Paris-Dauphine, CEREMADE, UMR CNRS 7534, PSL Research university
Place du Maréchal de Lattre de Tassigny 75016 Paris, France; clarkemove@gmail.com

Universidade Estadual de Campinas, Departamento de Matemática, IMECC
Campinas, Brazil; colivera@ime.unicamp.br

Abstract. We study the L^p-solutions for the semilinear heat equation with unbounded coefficients and driven by a infinite dimensional fractional Brownian motion with self-similarity parameter $H > 1/2$. Existence and uniqueness of local mild solutions are shown.

1. Introduction

The fractional Brownian motion, referred to as fBm in the sequel, due to its desirable properties of self-similarity and long-range dependence (among other features), has become quite a relevant stochastic process for mathematical modeling in engineering, mathematical finances, and natural sciences, to mention just a few. It was first introduced by Kolmogorov in [10], and later, the work of Mandelbrot and Van Ness [11] became a corner-stone that attracted the attention of researchers in the probabilistic community to this challenging object.

Nowadays, the study of ordinary and partial stochastic differential equations driven by a fractional noise is a very dynamic research topic, motivated by purely theoretical reasons and also by its variety of applications in the mathematical modeling of phenomena in physics, biology, hydrology, and other sciences. Besides, a special interest in the study of the existence and uniqueness of solutions to semilinear parabolic stochastic differential equations driven by an infinite-dimensional fractional noise has been recently developed (see for instance, Duncan, Pasik-Duncan and Maslowski [7]; Nualart and Vuillermot [15]; Maslowski and Schmalfuss [12], and Sanz-Sole and Vuillermot [18], and the references therein).

Other kind of driving noises have been also considered. In [3], Brzezniak, Neerven, Salopek, studied evolution equations with Liouville fractional Brownian motion; equations driven by Hermite or Rosenblatt process were addressed by Bonaccorsi and Tudor in [2], and Tudor in [20]. More recently, equations driven by Volterra noises were analysed by Coupek, Maslowski in [4] and by Coupek, Maslowski, and Ondrejat in [5].

In difference with the present manuscript, the articles [3], [4], [5] and [20] consider no non-linearity and deal only with linear equations, and the article [2] assumes that F is dissipative and has polynomial growth.

Analogously to deterministic partial differential equations, the first obstacle is the requirement of deciding which kind of solution concept will be considered, due to the

https://doi.org/10.5186/aasfm.2020.4505

2010 Mathematics Subject Classification: Primary 60H15, 60H30, 35R60, 35K05, 35K10, 35K58.

Key words: Stochastic partial differential equation, heat equation, mild solution, fractional Brownian motion, cylindrical fractional Brownian motion, unbounded coefficients.

variety of alternatives to choose. We address the study of existence and uniqueness of mild-solution to the initial value problem for the semilinear heat equation over a smoothly bounded open domain $U \subset \mathbf{R}^d$,

$$\begin{cases} \partial_t u(t) = \Delta u(t) + F(u(t)) + \partial_t B^H(t), \quad t \in [0,T], \\ u|_{t=0} = u_0, \end{cases} \tag{1}$$

In (1), F represents the nonlinear part of the equation, $u_0 \in L^p(U)$, and the random forcing field B^H is a Hilbert space-valued fractional Brownian motion defined on some complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$.

In this manuscript, the existence and uniqueness of local L^p-solutions for the stochastic parabolic equation (1) with unbounded parameter F and B^H a cylindrical fractional Brownian motion with selfsimilarity parameter $H > 1/2$, is proved. The approach to study L^p-solutions is based on the concept of mild solution, which can be obtained by rewriting (1) as an integral equation,

$$u(t) = S(t)u_0 + \int_0^t S(t-s)F(u(s))\,ds + \int_0^t S(t-s)\,dB^H(s),$$

and then proving that, in a suitable function space, the right-hand side defines a contraction.

Results on the existence of mild solutions with values in L^p were established by Giga in [8], Mazzucato in [13], and Weissler in [23] and [22] for the deterministic setting.

The rest of the manuscript is fashioned as follows. In Section 2 the basic concepts, hypothesis and tools are introduced. The results are presented in Section 3.

2. Preliminaries

Hypothesis, background and some useful notation are introduced in what follows. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a complete probability space.

2.1. Fractional Brownian motion. Let $T > 0$ be a fixed time horizon. Recall that a one-dimensional fractional Brownian motion $(b^H(t))_{t\in[0,T]}$ with Hurst parameter $H \in (0,1)$, is a centred Gaussian process with covariance function

$$\mathbf{E}\left[b^H(t)b^H(s)\right] = R_H(t,s) := \frac{1}{2}(t^{2H} + s^{2H} - |t-s|)^{2H}, \quad s,t \in [0,T]. \tag{2}$$

The fractional Brownian motion (*fBm*) can also be defined as the only self-similar Gaussian process with stationary increments.

Denote by $\mathcal{H}$ its associated canonical Hilbert space (reproducing kernel Hilbert space). If $H = \frac{1}{2}$ then $b^{\frac{1}{2}} = b$ is the standard Brownian motion (Wiener process) and in this case $\mathcal{H} = L^2([0,T])$. Otherwise $\mathcal{H}$ is the Hilbert space on $[0,T]$ extending the set of indicator functions $\mathbf{1}_{[0,t]}, t \in [0,T]$ by linearity and closure under the inner product

$$\left\langle \mathbf{1}_{[0,t]}; \mathbf{1}_{[0,s]} \right\rangle_{\mathcal{H}} = R_H(t,s)$$

As the fBm is a regular Volterra process only for $H > 1/2$, we will focus our analysis exclusively in this case. In order to define the concept of mild-solution through convolution integrals, we need to recall the definition of integrals with respect to the fBm. The followings facts will be needed in the sequel (we refer to [14] or [17] for their proofs):

• The fBm admits a representation as Wiener integral of the form

$$b^H(t) = \int_0^t K_H(t,s)\, db(s), \tag{3}$$

where $b = \{b(t), t \in [0,T]\}$ is a Wiener process, and $K_H(t,s)$ is the kernel

$$K_H(t,s) = c_H s^{\frac{1}{2}-H} \int_s^t (u-s)^{H-\frac{3}{2}} u^{H-\frac{1}{2}}\, du \tag{4}$$

where $t > s$ and $c_H = \left(\frac{H(2H-1)}{\beta(2-2H, H-\frac{1}{2})}\right)^{\frac{1}{2}}$, where β is the *Beta* function.

• For every $s < T$, consider the operator $K_H^*\colon \mathcal{H} \mapsto L^2([0,T])$, defined by

$$(K_H^* \phi)(s) = \int_s^T \phi(t)(s) \frac{\partial K_H}{\partial t}(t,s)\, dt. \tag{5}$$

Notice that, $\left(K_H^* \phi \mathbf{1}_{[0,t]}\right)(s) = K_H(t,s)\phi(s)\mathbf{1}_{[0,t]}(s)$, and the operator K_H^* is an isometry between $\mathcal{H}$ and $L^2([0,T])$ (see [1] or [14]). Hence, for every $\phi \in \mathcal{H}$ it is possible to establish the following relationship between a Wiener integral with respect to the fBm and a Wiener integral with respect to the standard Brownian motion b

$$\int_0^t \phi(s)\, db^H(s) = \int_0^t \left(K_H^* \phi\right)(s)\, db(s), \tag{6}$$

for every $t \in [0,T]$ and $\phi \mathbf{1}_{[0,t]} \in \mathcal{H}$ if and only if $K_H^* \phi \in L^2([0,T])$.

In general, the existence of the right-hand side of (6) requires careful justification (see [14, Section 5.1]). As we will work only with Wiener integrals over Hilbert spaces, we point out that if X is a Hilbert space and $f \in L^2([0,T]; X)$ is a deterministic function, then relation (6) holds, and the right hand-side is well defined in $L^2(\Omega; X)$ if $K_H^* f$ is in $L^2([0,T] \times X)$.

2.2. Cylindrical fractional Brownian motion. As in [7] or [19], we define the standard *cylindrical* fractional Brownian motion in X as the formal series

$$B^H(t) = \sum_{n=0}^{\infty} e_n b_n^H(t), \tag{7}$$

where $\{e_n, n \in \mathbf{N}\}$ is a complete orthonormal basis in X. It is well known that the infinite series (7) does not converge in $L^2(\mathbf{P})$, hence $B^H(t)$ is not a well-defined X-valued random variable. Nevertheless, for every Hilbert space X_1 such that $X \hookrightarrow X_1$, the linear embedding is a Hilbert–Schmidt operator, therefore, the series (7) defines a X_1-valued random variable and $\{B^H(t), t \geq 0\}$ is a X_1-valued cylindrical fBm.

Following the approach for a cylindrical Brownian motion introduced in [6], it is possible to define a stochastic integral of the form

$$\int_0^T f(t)\, dB^H(t), \tag{8}$$

where $f\colon [0,T] \mapsto \mathcal{L}(X,Y)$ and Y is another real and separable Hilbert space, and the integral (8) is a Y-valued random variable that is independent of the choice of X_1.

Let f be a deterministic function with values in $\mathcal{L}_2(X,Y)$, the space of Hilbert-Schmidt operators from X to Y. We consider the following assumptions on f.

i.- For each $x \in X$, $f(\cdot)x \in L^p([0,T]; Y)$, for $p > 1/H$.

ii.- $\alpha_H \int_0^T \int_0^T |f(s)|_{\mathcal{L}_2(X,Y)}\, |f(t)|_{\mathcal{L}_2(X,Y)} |s-t|^{2H-2} ds\, dt \ < \infty$.

The stochastic integral (8) is defined as

$$\int_0^t f(s)\, dB^H(s) := \sum_{n=1}^{\infty} \int_0^t f(s) e_n \, db_n^H(s) = \sum_{n=1}^{\infty} \int_0^t (K_H^* f e_n)(s)\, db_n(s), \tag{9}$$

where b_n is the standard Brownian motion linked to the fBm b_n^H via the representation formula (3). Since $fe_n \in L^2([0,T];Y)$ for each $n \in \mathbf{N}$, the terms in the series (9) are well defined. Besides, the sequence of random variables $\left\{\int_0^t fe_n\, db_n^H\right\}$ are mutually independent (see [7]).

The series (9) is finite if

$$\sum_n \|K_H^*(fe_n)\|^2_{L^2([0,T];V)} = \sum_n \| \, \|fe_n\|_{\mathcal{H}} \, \|_V^2 < \infty. \tag{10}$$

If we consider $X = Y = \mathcal{H}$, we have

$$\begin{aligned}\sum_{n=1}^{\infty} \int_0^t f(s) e_n \, db_n^H(s) &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e_m \int_0^t \langle f(s)e_n, e_m\rangle_{\mathcal{H}} \; db_n^H(s) \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} e_m \int_0^t \langle K_H^*(f(s)e_n), e_m\rangle_{\mathcal{H}} \; db_n(s) \\ &= \sum_{n=1}^{\infty} \int_0^t K_H^*(f(s)e_n)\, db_n(s).\end{aligned} \tag{11}$$

2.3. Semigroup. It is well known that the Laplacian Δ is the infinitesimal generator of an analytic, strongly continuous semi-group of linear operators $(S(t), t \geq 0)$ acting on $L^p(U)$ and given by $S(t) = e^{-t\Delta}$. Besides, for bounded domains the following estimate holds (see [21])

$$\|S(t)u\|_p \leq \frac{1}{t^{\frac{d}{2}(1/r-1/p)}} \|u\|_r, \quad \text{for } 1 < r \leq p < \infty. \tag{12}$$

3. Results

In this section we study the parabolic problem (1) in the space $L^p(U)$. The required hypothesis are introduced as well as the notion of mild-solution.

3.1. Hypothesis. We assume that F is a nonlinear mapping from $L^p(U)$ onto $L^m(U)$ such that $F(0) = 0$, and for some $\alpha > 0$ and $m = \frac{p}{1+\alpha}$, the estimate

$$\|F(u) - F(v)\|_m \leq C\|u - v\|_p(\|u\|_p^\alpha + \|v\|_p^\alpha) \tag{13}$$

holds, with C a positive constant.

In addition, the initial condition satisfies

$$u_0 \in L^p(U). \tag{14}$$

Besides, the cylindrical fBm B^H has selfsimilarity parameter $H > 1/2$ and

$$H > d/4, \; p \cdot H \geq 1, \; \text{and } 2p > \alpha d. \tag{15}$$

3.2. Mild-solution. Within the framework of paragraph 2.2 we consider $X = L^2(U)$, $f = S(t - \cdot)$ and the complete orthonormal basis $\{e_n\}_{n \in \mathbf{N}}$ of eigenfunctions of the Laplacian operator, the stochastic convolution is given by

$$\int_0^t S(t-s)\,dB^H(s) = \sum_{j=1}^{\infty} \int_0^t S(t-s)e_j\,d\beta_j^H(s).$$

Consider the mild formulation of equation (1) (see [7])

$$u(t) = S(t)u_0 + \int_0^t S(t-s)F(u(s))\,ds + \int_0^t S(t-s)\,dB^H(s). \tag{16}$$

Definition 3.1. A measurable function $u\colon \Omega \times [0,T] \mapsto L^p(U)$ is a mild solution of the equation (1) if

(1) u satisfies the mild formulation (16) with probability one.
(2) $u \in C([0,T], L^p(U))$.

Definition 3.2. Let T_0 be a stopping time. A measurable function $u\colon \Omega \times [0,T] \to L^p(U)$ is a local mild solution of (1) in $C([0,T_0], L^p(U))$ with stopping time $T_0 > 0$, if it satisfies Definition 3.1 on $[0,T_0]$. It is the unique local mild solution with stopping time T_0, if two solutions are modifications of each other on $[0,T_0]$.

3.3. Existence. Consider the linear problem

$$\begin{cases} \partial_t z(t) = \Delta z(t) + \partial_t B_t^H, & t \in [0,T], \\ z|_{t=0} = 0, \end{cases} \tag{17}$$

whose mild solution is given by

$$z(t) = \int_0^t S(t-s)\,dB^H(s).$$

Denote

$$K_0 := \max\left\{ \|u_0\|_p, \sup_{t\in[0,T]} \left\| \int_0^t S(t-s)dB^H(s) \right\|_p \right\} = \max\left\{ \|u_0\|_p, \sup_{t\in[0,T]} \|z(t)\|_p \right\},$$

$$\tilde{C}(t) = \begin{cases} C\ \frac{t^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}}\ (6K_0)^{\alpha}, & \text{if } \alpha \geq \frac{\ln(3)}{\ln(2)}, \\ C\ \frac{t^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}}\ (3K_0)^{\alpha+1}, & \text{if } \alpha < \frac{\ln(3)}{\ln(2)}, \end{cases}$$

and define

$$T_0 = \begin{cases} T, & \text{if } \tilde{C}(T) < 1, \\ \inf\{0 \leq t \leq T\colon \tilde{C}(t) \geq 1\}, & \text{if } \tilde{C}(T) \geq 1. \end{cases} \tag{18}$$

Theorem 3.3. *Assume hypothesis (13), (14), (15). Then there exists a local mild solution $u \in C([0,T_0], L^p(U))$.*

Proof. Since $H > \frac{d}{4}$ and $pH \geq 1$, the results in [5] allow us to conclude that the mild solution z to the linear problem (17) is in $C([0,T], L^p(U))$. Therefore,

$$\sup_{t\in[0,T]} \left\| \int_0^t S(t-s)\,dB^H(s) \right\|_p < \infty.$$

Now, in order to construct a contraction that will allow us to use a fix point argument, let us assume that $\|u\|_{C([0,T_0],L^p(U))} := \sup_{t\in[0,T_0]} \|u(t)\|_p \leq 3K_0$. Set

$$G[u](t) := S(t)u_0 + \int_0^t S(t-s)F(u(s))\,ds + z(t).$$

We shall show that $\sup_{t\in[0,T_0]} \|G[u](t)\|_p \leq 3K_0$. We have

$$\|G[u](t)\|_p \leq \|S(t)u_0\|_p + \int_0^t \|S(t-s)F(u(s))\|_p \, ds + \|z(t)\|_p.$$

As $(S(t))_{t\geq 0}$ is a semigroup of contractions, for every $t \geq 0$

$$\|S(t)u_0\|_p \leq \|u_0\|_p, \tag{19}$$

and

$$\begin{aligned} \int_0^t \|S(t-s)F(u(s))\|_p \, ds &\leq \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \|F(u(s))\|_{\frac{p}{\alpha+1}} \, ds \\ &\leq C \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \|u(s)\|_p^{\alpha+1} \, ds, \end{aligned} \tag{20}$$

where we used (12) and hypothesis (13).

From (19) and (20) we deduce that

$$\begin{aligned} \|G[u](t)\|_p &\leq 2K_0 + C \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \|u(s)\|_p^{\alpha+1} \, ds \\ &\leq 2K_0 + C \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \left(\sup_{s\in[0,T_0]} \|u(s)\|_p \right)^{\alpha+1} ds \\ &\leq 2K_0 + C(3K_0)^{\alpha+1} \frac{t^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}}. \end{aligned}$$

Hence,

$$\begin{aligned} \sup_{[0,T_0]} \|G[u](t)\|_p &\leq 2K_0 + C(3K_0)^{\alpha+1} \frac{T_0^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}} \\ &= 3K_0 \left(\frac{2}{3} + \ C \frac{T_0^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}} \, (3K_0)^{\alpha} \right) \leq 3K_0, \end{aligned}$$

whenever

$$C \, \frac{T_0^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}} \, (3K_0)^{\alpha+1} < 1. \tag{21}$$

We shall show now that $G\colon X \mapsto X$ is a contraction, where $X := \{u \in C([0,T_0], L^p(U))\colon \|u\|_{C([0,T_0],L^p(U))} \leq 3K_0\}$. Let Fix $u, v \in X$ then $t \in [0,T_0]$, we have

$$\begin{aligned} \|G[u](t) - G[v](t)\|_p &\leq \int_0^t \|S(t-s)\left(F(u(t)) - F(v(t))\right)\|_p \, ds \\ &\leq \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \|F(u(t)) - F(v(t))\|_{\frac{p}{\alpha+1}} \, ds \\ &\leq C \int_0^t (t-s)^{-\frac{d\alpha}{2p}} \|u(t)-v(t)\|_p \left(\|u(t)\|_p^{\alpha} + \|v(t)\|_p^{\alpha} \right) \, ds \\ &\leq C(6K_0)^{\alpha} \, \frac{T_0^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}} \, \sup_{t\in[0,T_0]} \|u(t)-v(t)\|_p \, ds, \end{aligned} \tag{22}$$

where we used (12) and hypothesis (13). Hence, if

$$C\,\frac{T_0^{1-\frac{d\alpha}{2p}}}{1-\frac{d\alpha}{2p}}\,(6K_0)^{\alpha}<1, \tag{23}$$

then

$$\sup_{t\in[0,T_0]}\|G[u](t)-G[v](t)\|_p<\sup_{t\in[0,T_0]}\|u(t)-v(t)\|_p.$$

Therefore, G is a contraction. Hence, there exist a unique fixed point. □

3.4. Example of a non-linearity F. An example of a non-linearity F satisfying condition (13) is as follows. Let f be a mapping from $\mathbf{R}^d$ to $\mathbf{R}^d$ verifying $f(0)=0$ and

$$|f(y)-f(x)|\leq C|x-y|(|x|^{\alpha}+|y|^{\alpha}),$$

for $\alpha>0$.

Set $F(u)(x)=f(u(x))$, hence, by Hölder's inequality F satisfies (13). As an especific example to construct the non-linearity F, we may consider the function $f(x)=x|x|^{\alpha}$.

Remark 3.4. The results presented in the manuscript can be generalized to following setting: X a real separable Hilbert space, and (D,μ) be a measure space.

For $1\leq p<\infty$, $L^p=L^p(D,\mu)$ is a separable Banach space. We consider the following stochastic differential equation

$$\begin{cases}\partial_t u(t)=Au(t)+F(u(t))+\Phi\partial_t B_t^H, & t\in[0,T],\\ u|_{t=0}=u_0,\end{cases} \tag{24}$$

where $u_0\in L^p$, $A\colon Dom(A)\subset L^p\mapsto L^p$, is the infinitesimal generetor of an analytic strongly continuous semigroup of linear operators $(S(t),t\geq 0)$ acting on L^p, and $\Phi\in\gamma(X,L^p)$ where $\gamma(X,L^p)$ denote the space of the γ-radonifying operator(see [16]). Under similar conditions as (12), (13) and (15), and assuming that for $\lambda\in[0,H)$, $\|S(t)\Phi\|_{\gamma(X,L^p)}\leq t^{-\lambda}$, by following the same steps as in the proof of Theorem 3.3 and Corollary 4.3 in [5], is possible to show the existence of an unique local mild solution to (24) in $C([0,T],L^p)$. The conditions on Φ allows to consider both $\Phi=Id$ (that corresponds to noise that is white in space) or $\Phi\in\gamma(X,L^p)$ (that corresponds to correlated noise in space).

Acknowledgements. Christian Olivera is partially supported by FAPESP by the grants 2017/17670-0 and 2015/07278-0. Also supported by CNPq by the grant 426747/2018-6. The authors would also like to thank the reviewers comments, which enriched the content of the article.

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Received 3 August 2018 • Received 14 February 2019 • Accepted 24 May 2019