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ON GENERALIZED RESOLVENTS OF SYMMETRIC LINEAR RELATIONS IN A PONTRJAGIN SPACE

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Introduction

In [4] we gave a characterization of the generalized resolvents of a symmetric operator with arbitrary defect numbers in a Pontrjagin space $\Pi_\varkappa$. The purpose of this note is to extend this and related results to symmetric linear relations in $\Pi_\varkappa$. As was pointed out in [5], the need for this kind of extension arises e.g. in connection with differential relations with an indefinite weight function.

Because this paper is a continuation of [5], we shall freely use the notions and results from [5].

1. Generalized resolvents

Throughout this paper $\mathfrak{H}$ denotes a Pontrjagin space $\Pi_\varkappa$ with an (indefinite) inner product $[\cdot|\cdot]$ which has $\varkappa$ negative squares, and $\mathfrak{H}^2$ is the product space $\mathfrak{H}\oplus\mathfrak{H}$. Furthermore, T always stands for a closed symmetric linear relation in $\mathfrak{H}$; i.e., T is a closed subspace of $\mathfrak{H}^2$ with $T\subset T^+$, where

$$T^+ := \{(h,k)\in\mathfrak{H}^2 | [g|h] = [f|k] \text{ for all } (f,g)\in T\}.$$

A self-adjoint extension S of T, i.e. $S^+=S\supset T$, is said to be *regular* if $S\subset\mathfrak{K}^2$, where $\mathfrak{K}\supset\mathfrak{H}$ is a Pontrjagin space with $\varkappa$ negative squares. Let S be such an extension. The function $R\colon \varrho(S)\to\mathscr{B}(\mathfrak{H})$,

$$R(z) := P(S-zI)^{-1}|_{\mathfrak{H}} \qquad (z\in\varrho(S)),$$

is called a *generalized resolvent* of T; here P denotes the orthogonal projector of $\mathfrak{K}$ onto $\mathfrak{H}$. If in addition S extends a maximal symmetric relation T' in $\mathfrak{H}$ with the upper defect number $\mathfrak{n}_+(T')=0$ (resp. lower defect number $\mathfrak{n}_-(T')=0$), then R is *upper canonical* (resp. *lower canonical*).

We suppose that the domain $\mathfrak{D}(T)$ of T includes the negative component $\mathfrak{H}_-$ of a fundamental decomposition of $\mathfrak{H}$. Then T has regular self-adjoint extensions ([5], Corollary 4.7), and there exists a constant $c>0$ such that the spaces $\mathfrak{N}_z:=\mathfrak{R}(T-\bar{z}I)^\perp$ with $|\operatorname{Im} z|>c$ are Hilbert spaces with respect to the indefinite inner product $[\cdot|\cdot]$; see [5], Theorem 4.10.

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Take a fixed complex number w with $\operatorname{Im} w > c$ and denote by V the Cayley transform $C_w(T)$ of T:

$$V := \{(g - wf,\, g - \bar{w}f) \mid (f, g) \in T\}.$$

Then $\mathfrak{N}_{\bar{w}} = \mathfrak{D}(V)^{\perp}$ and $\mathfrak{N}_w = \mathfrak{R}(V)^{\perp}$; see [5]. Let Γ_0 (resp. Γ_∞) be the orthogonal projector of $\mathfrak{H}$ onto $\mathfrak{N}_{\bar{w}}$ (resp. $\mathfrak{N}_w$). The characteristic function of V is defined by the equation

$$X(\lambda) := \lambda^{-1}\Gamma_0(I - \lambda V')^{-1}|_{\mathfrak{N}_w} \qquad (|\lambda| < 1);$$

here V' is the zero extension of V:

$$V'f := \begin{cases} Vf & \text{for } f \in \mathfrak{D}(V), \\ 0 & \text{for } f \in \mathfrak{N}_{\bar{w}}. \end{cases}$$

Then the *characteristic function* of T is $Y(z) := X(\lambda(z)^{-1})$, where $\lambda(z) := (z - \bar{w})/(z - w)$ for z in the complex plane $\boldsymbol{C}$. Note that Y is a meromorphic function in the open upper half-plane $\boldsymbol{C}_+$ of $\boldsymbol{C}$ with values in $\mathscr{B}(\mathfrak{N}_w, \mathfrak{N}_{\bar{w}})$; see [2] or [4].

Let S be a fixed regular self-adjoint extension of T and R the corresponding generalized resolvent. For $z \in \varrho(S)$ we define (see [2])

$$\Gamma_+(z) := \Gamma_\infty + (z - w)R(z)\Gamma_\infty,$$

$$\Gamma_-(z) := \Gamma_0 + (z - \bar{w})R(z)\Gamma_0,$$

$$F(z) := (z - w)^{-1}\Gamma_\infty\{(z - \bar{w})I - (w - \bar{w})[I + (z - w)R(z)]^{-1}\}|_{\mathfrak{N}_{\bar{w}}}.$$

Then F belongs to the class $\mathscr{K}_+(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$ of the functions which are holomorphic in $\boldsymbol{C}_+$ with contractive operator values in $\mathscr{B}(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$. In the following we shall use the phrase "for almost all $z \in \boldsymbol{C}_+$" or shortly "for a.a. $z \in \boldsymbol{C}_+$" to mean "for all $z \in \boldsymbol{C}_+$ with the possible exception of a countable set which does not have any cluster points in $\boldsymbol{C}_+$".

Theorem 1.1. *Let T be a closed symmetric linear relation in a Pontrjagin space $\mathfrak{H}$ with $\mathfrak{D}(T) \supset \mathfrak{H}_-$ and $\mathfrak{n}_\pm(T) > 0$ and let w be a complex number with* $\operatorname{Im} w > c$. *If R is a given regular generalized resolvent of T, then the formula*

$$\tilde{R}(z) = R(z) + (w - \bar{w})^{-1}\Gamma_+(z)B(z)\Gamma_-(\bar{z})^+, \tag{1.1}$$

where

$$B(z) := (I - F(z)Y(z))(I - \tilde{F}(z)Y(z))^{-1}(\tilde{F}(z) - F(z)) \tag{1.2}$$

for almost all $z \in \boldsymbol{C}_+$, defines a bijective correspondence between all regular generalized resolvents $\tilde{R}$ of T and all functions $\tilde{F} \in \mathscr{K}_+(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$.

Furthermore, $\tilde{R}$ is upper (resp. lower) canonical if and only if $\tilde{F}$ is independent of z and $\tilde{F}(z)$ (resp. $\tilde{F}(z)^+$) is isometric.

This result extends [2], Satz 4.2. The proof follows in the same way as in the operator case via the Cayley transformation; see [2] and [4].

In the case of equal defect numbers the characterization of the generalized resolvents of a symmetric linear operator or relation uses the so-called Q-function instead of the characteristic function Y and dissipative operators or relations instead of contractive operators $\tilde{F}(z)$; see [1] and [3]. We show that this kind of characterization can be given by use of Theorem 1.1.

For this, let T be as in Theorem 1.1 and suppose that the defect numbers of T are both equal to $\mathfrak{n}$. In this case T has a self-adjoint extension in the original space $\mathfrak{H}$. Indeed, the Cayley transform $V=C_w(T)$ of T is an isometric operator with $\dim \mathfrak{D}(V)^\perp = \dim \mathfrak{R}(V)^\perp = \mathfrak{n}$ and $\mathfrak{D}(V)^\perp$ as well as $\mathfrak{R}(V)^\perp$ are Hilbert spaces, so that V has a unitary extension in $\mathfrak{H}$. The inverse Cayley transform of this unitary operator is then a self-adjoint extension of T in $\mathfrak{H}$.

Choose a self-adjoint extension S of T in $\mathfrak{H}$ and let R be the corresponding generalized resolvent. Then, by Theorem 1.1, the function $F\in\mathscr{K}_+(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$ is independent of z and $FF^+=F^+F$ with $F:=F(z)$.

To form a Q-function of T we first choose a Hilbert space $\mathfrak{G}$ with $\dim\mathfrak{G}=\mathfrak{n}$ and a bijective operator $\Gamma\in\mathscr{B}(\mathfrak{G}, \mathfrak{N}_w)$. Define

$$Q'(z) := \begin{cases} (C_{-iy})^{-1}(FY(z)) & \text{for } z\in \boldsymbol{C}_+, \\ Q'(\bar{z})^+ & \text{for } z\in \boldsymbol{C}_-; \end{cases}$$

here $y:=\operatorname{Im} w$. It is not too difficult to show that the functions $Q(z):=\Gamma^+Q'(z)\Gamma\in\mathscr{B}(\mathfrak{G})$ and $\Gamma(z):=\Gamma_+(z)\Gamma\in\mathscr{B}(\mathfrak{G}, \mathfrak{N}_z)$ satisfy the equation

$$(z-\bar{\zeta})^{-1}(Q(z)-Q(\zeta)^+) = \Gamma(\zeta)^+\Gamma(z) \qquad (z, \zeta\in\varrho(S)),$$

i.e., Q is a Q-function of T in the sense of [1].

Let $\tilde{R}$ be an arbitrary regular generalized resolvent of T and $\tilde{F}\in\mathscr{K}_+(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$ the function assigned to it by Theorem 1.1. With the function B given in (1.2) we get

$$\begin{aligned}(1.3)\qquad & (w-\bar{w})^{-1}B(z)F^+ \\ &= (2iy)^{-1}\{(I-FY(z))^{-1}-\tilde{F}(z)Y(z)(I-FY(z))^{-1}\}^{-1}(\tilde{F}(z)F^+-I) \\ &= -\{Q'(z)+iyI-\tilde{F}(z)F^+(Q'(z)-iyI)\}^{-1}(I-\tilde{F}(z)F^+).\end{aligned}$$

Define $D'(z):=(C_{-iy})^{-1}(\tilde{F}(z)F^+)$; then a little calculation gives

$$(1.4)\qquad Q'(z)+D'(z) = (I-\tilde{F}(z)F^+)\{Q'(z)+iyI-\tilde{F}(z)F^+(Q'(z)-iyI)\}.$$

Furthermore, one can verify that $\Gamma_-(\bar{z})^+=F^+(\Gamma^+)^{-1}\Gamma(\bar{z})^+$. Put this and (1.3)—(1.4) together to get

$$\begin{aligned}(w-\bar{w})^{-1}\Gamma_+(z)B(z)\Gamma_-(\bar{z})^+ &= (2iy)^{-1}\Gamma(z)\Gamma^{-1}B(z)F^+(\Gamma^+)^{-1}\Gamma(\bar{z})^+ \\ &= -\Gamma(z)(Q(z)+D(z))^{-1}\Gamma(\bar{z})^+,\end{aligned}$$

where $D(z):=\Gamma^+D'(z)\Gamma$.

Denote by $\mathscr{D}_+(\mathfrak{G})$ the set of all functions $z\mapsto D(z)$ such that $D(z)$, $z\in\boldsymbol{C}_+$, is a maximal dissipative linear relation in $\mathfrak{G}$ and the mapping $z\mapsto C_{-iy}(D(z))$ is holomorphic in C_+. By using the results of [5] one can show that $D\in\mathscr{D}_+(\mathfrak{G})$. As the calculations above are invertible we can write the following result, which extends [1], Theorem 5.1 and [3], Theorem 3.2.

Corollary 1.2. *Let T be a closed symmetric linear relation in a Pontrjagin space $\mathfrak{H}$ with $\mathfrak{D}(T)\supset\mathfrak{H}_-$ and $\mathfrak{n}_+(T)=\mathfrak{n}_-(T)>0$. Let R be a generalized resolvent of T in the original space. The formula*

$$\tilde{R}(z) = R(z)-\Gamma(z)(Q(z)+D(z))^{-1}\Gamma(\bar{z})^+$$

gives a bijective correspondence between the set of all regular generalized resolvents $\tilde{R}$ of T and the set $\mathscr{D}_+(\mathfrak{G})$.

2. Resolvent matrices

In this section we suppose that T is a closed symmetric linear relation in a Pontrjagin space $\mathfrak{H}$ with $\mathfrak{D}(T)\supset\mathfrak{H}_-$ and $\mathfrak{n}_+(T)\geqq\mathfrak{n}_-(T)>0$. Furthermore, let w be a fixed complex number with $\operatorname{Im} w>c$.

Let us take two closed subspaces $\mathfrak{L}_\pm$ of $\mathfrak{H}$ with $\dim\mathfrak{L}_\pm=\mathfrak{n}_\pm(T)$. If $P_\pm$ are (not necessarily orthogonal) projectors of $\mathfrak{H}$ onto $\mathfrak{L}_\pm$, then the adjoints $P_\pm^+$ are also projectors and $\mathfrak{L}_\pm^+:=\mathfrak{R}(P_\pm^+)$ are closed subspaces. The set of all operator matrices

$$\mathscr{W}=\begin{bmatrix} W_{11} & W_{12}\\ W_{21} & W_{22}\end{bmatrix}$$

with $W_{11}\in\mathscr{B}(\mathfrak{N}_w,\mathfrak{L}_-^+)$, $W_{12}\in\mathscr{B}(\mathfrak{N}_{\bar{w}},\mathfrak{L}_-^+)$, $W_{21}\in\mathscr{B}(\mathfrak{N}_w,\mathfrak{L}_+)$ and $W_{22}\in\mathscr{B}(\mathfrak{N}_{\bar{w}},\mathfrak{L}_+)$ is denoted by $\mathscr{B}_2(\mathfrak{N}_w,\mathfrak{N}_{\bar{w}};\mathfrak{L}_-^+,\mathfrak{L}_+)$. For W in this set we can define in a natural way the inverse $\mathscr{W}^{-1}\in\mathscr{B}_2(\mathfrak{L}_-^+,\mathfrak{L}_+;\mathfrak{N}_w,\mathfrak{N}_{\bar{w}})$, if it exists, and the adjoint $\mathscr{W}^+\in\mathscr{B}_2(\mathfrak{L}_-,\mathfrak{L}_+^+;\mathfrak{N}_w,\mathfrak{N}_{\bar{w}})$:

$$\mathscr{W}^+:=\begin{bmatrix} W_{11}^+ & W_{21}^+\\ W_{12}^+ & W_{22}^+\end{bmatrix}.$$

Furthermore, we denote by $M_{\mathscr{W}}$ the "Möbius transformation" induced by $\mathscr{W}$:

$$M_{\mathscr{W}}(F) := (W_{11}F+W_{12})(W_{21}F+W_{22})^{-1}$$

for $F\in\mathscr{B}(\mathfrak{N}_{\bar{w}},\mathfrak{N}_w)$. For the basic properties of $M_{\mathscr{W}}$, see [2].

If $\tilde{R}$ is a regular generalized resolvent of T, then the function $z\mapsto\tilde{\Omega}(z):= P_-^+\tilde{R}(z)|_{\mathfrak{L}_+}$ with values in $\mathscr{B}(\mathfrak{L}_+,\mathfrak{L}_-^+)$ is called a *(P_+,P_-)-resolvent* of T. These resolvents are best studied by means of the so-called (P_+,P_-)-resolvent matrices. To define the latter, we denote by $\varrho(\mathfrak{L}_+,\mathfrak{L}_-)$ the set of all $z\in C_+$ for which

$$\mathfrak{R}(T-zI)\dotplus\mathfrak{L}_+ = \mathfrak{R}(T-\bar{z}I)\dotplus\mathfrak{L}_- = \mathfrak{H}.$$

A matrix function $\mathscr{W}$ is called a (P_+, P_-)*-resolvent matrix* for T if it has the following properties:

1) $\mathscr{W}$ is defined for a.a. $z\in\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$, has values in $\mathscr{B}_2(\mathfrak{N}_w, \mathfrak{N}_{\bar{w}}; \mathfrak{L}_-^+, \mathfrak{L}_+)$ and is meromorphic;

2) $\mathscr{W}(z)^{-1}\in\mathscr{B}_2(\mathfrak{L}_-^+, \mathfrak{L}_+; \mathfrak{N}_w, \mathfrak{N}_{\bar{w}})$ exists for a.a. $z\in\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$;

3) $M_{\mathscr{W}(z)}(F)$ is an operator for all contractive operators $F\in\mathscr{B}(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$ and for a.a. $z\in\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$;

4) the formula

$$\tilde{\Omega}(z) = M_{\mathscr{W}(z)}(\tilde{F}(z)) \qquad \text{for a.a. } z\in\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$$

gives a bijective mapping between the set of all (P_+, P_-)-resolvents $\tilde{\Omega}$ of T and the set of all $\tilde{F}\in\mathscr{K}_+(\mathfrak{N}_{\bar{w}}, \mathfrak{N}_w)$.

The existence of a (P_+, P_-)-resolvent matrix is settled by the following result, which generalizes [2], Satz 5.2.

Theorem 2.1. *Let* T *be a closed symmetric linear relation in a Pontrjagin space* $\mathfrak{H}$ *with* $\mathfrak{D}(T)\supset\mathfrak{H}_-$ *and* $\mathfrak{n}_+(T)\geqq\mathfrak{n}_-(T)>0$. *Let* $\mathfrak{L}_\pm$ *be closed subspaces of* $\mathfrak{H}$ *such that* $\dim\mathfrak{L}_\pm=\mathfrak{n}_\pm(T)$ *and* $\varrho(\mathfrak{L}_+, \mathfrak{L}_-)\neq\emptyset$. *If* $P_\pm$ *are projectors onto* $\mathfrak{L}_\pm$, *then* T *has a* (P_+, P_-)*-resolvent matrix.*

Proof. Choose the R in Theorem 1.1 to be lower canonical. Then with some manipulation one can put (1.1) in the form

$$\tilde{\Omega}(z) = P_-^+\tilde{R}(z)|_{\mathfrak{L}_+} = M_{\mathscr{W}(z)}(F(z)),$$

where the components of the desired (P_+, P_-)-resolvent matrix $\mathscr{W}$ are given by

$$\begin{aligned} W_{11}(z) &:= -P_-^+R(z)Z(z)^{-1}Y(z)+(w-\bar{w})^{-1}P_-^+\Gamma_+(z)|_{\mathfrak{N}_w},\\ W_{12}(z) &:= P_-^+R(z)Z(z)^{-1}-(w-\bar{w})^{-1}P_-^+\Gamma_+(z)F(z),\\ W_{21}(z) &:= -Z(z)^{-1}Y(z),\\ W_{22}(z) &:= Z(z)^{-1} \end{aligned} \tag{2.1}$$

with

$$Z(z) := (I-Y(z)F(z))\Gamma_-(\bar{z})^+|_{\mathfrak{L}_+}.$$

For details see [2] and [4], where the operator case is considered.

We proceed to characterize all the (P_+, P_-)-resolvent matrices. For this define

$$\mathscr{J} := \begin{bmatrix} I & O\\ O & -I\end{bmatrix}\in\mathscr{B}_2(\mathfrak{N}_w, \mathfrak{N}_{\bar{w}}; \mathfrak{N}_w, \mathfrak{N}_{\bar{w}})$$

and denote by $P(z)$, $z\in\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$, the projector onto $\mathfrak{L}_+$ along $\mathfrak{R}(T-zI)$. This P has a representation

$$P(z) = (\Gamma_-(\bar{z})^+|_{\mathfrak{L}_+})^{-1}\Gamma_-(\bar{z})^+,$$

which implies that P and Q,

$$Q(z) := P_-^+R(z)(I-P(z)),$$

are meromorphic in $\varrho(\mathfrak{L}_+, \mathfrak{L}_-)$; see [2].

The same lines of reasoning as in the operator case show that the (P_+, P_-)-resolvent matrix $\mathscr{W}$ with components (2.1) satisfies the equation

$$(2.2)\qquad (w-\bar{w})\mathscr{W}(z)\mathscr{J}\mathscr{W}(\zeta)^+ = \mathscr{X}(z,\zeta)\mathscr{J} \qquad \text{for a.a. } z\in\varrho(\mathfrak{L}_+,\mathfrak{L}_-),$$

where the matrix function $\mathscr{X}$ is given by

$$\begin{aligned} X_{11}(z,\zeta) &:= P_-^+(Q(z)-Q(\zeta)^+)|_{\mathfrak{L}_-}-(z-\bar{\zeta})Q(z)Q(\zeta)^+|_{\mathfrak{L}_-},\\ X_{12}(z,\zeta) &:= -P_-^+P(\zeta)^+|_{\mathfrak{L}_+^+}-(z-\bar{\zeta})Q(z)P(\zeta)^+|_{\mathfrak{L}_+^+},\\ X_{21}(z,\zeta) &:= -P(z)|_{\mathfrak{L}_-}+(z-\bar{\zeta})P(z)Q(\zeta)^+|_{\mathfrak{L}_-},\\ X_{22}(z,\zeta) &:= (z-\bar{\zeta})P(z)P(\zeta)^+|_{\mathfrak{L}_+^+}. \end{aligned}$$

Note that (2.2) can be written in the form

$$\begin{aligned}(\bar{w}-w)\mathscr{W}(z)\mathscr{J}\mathscr{W}(\zeta)^+ &= (z-\bar{\zeta})\begin{bmatrix} Q(z)\\ -P(z)\end{bmatrix}\begin{bmatrix} Q(\zeta)^+|_{\mathfrak{L}_-} & -P(\zeta)^+|_{\mathfrak{L}_+^+}\end{bmatrix}\\ &\quad+\begin{bmatrix} P_-^+(Q(\zeta)^+-Q(z))|_{\mathfrak{L}_-} & -P_-^+P(\zeta)^+|_{\mathfrak{L}_+^+}\\ P(z)|_{\mathfrak{L}_-} & O\end{bmatrix}.\end{aligned}$$

Reasoning further as in [2] (see also [4]) we derive the following characterizations of the (P_+, P_-)-resolvent matrices.

Theorem 2.2. *Let the assumptions of Theorem 2.1 be fulfilled and let* $\tilde{\mathscr{W}}\colon \varrho(\mathfrak{L}_+,\mathfrak{L}_-)\to\mathscr{B}_2(\mathfrak{N}_w,\mathfrak{N}_{\bar{w}};\mathfrak{L}_-^+,\mathfrak{L}_+)$ *be meromorphic. Then the following facts are equivalent:*

(i) $\tilde{\mathscr{W}}$ *is a* (P_+, P_-)*-resolvent matrix of* T;

(ii) *there exists a matrix* $\mathscr{U}\in\mathscr{B}_2(\mathfrak{N}_w,\mathfrak{N}_{\bar{w}};\mathfrak{N}_w,\mathfrak{N}_{\bar{w}})$ *such that* $\mathscr{U}\mathscr{J}\mathscr{U}^+=\mathscr{J}$ *and* $\tilde{\mathscr{W}}(z)=\mathscr{W}(z)\mathscr{U}$ *for a.a.* $z\in\varrho(\mathfrak{L}_+,\mathfrak{L}_-)$;

(iii) $\tilde{\mathscr{W}}(z)^{-1}\in\mathscr{B}_2(\mathfrak{L}_-^+,\mathfrak{L}_+;\mathfrak{N}_w,\mathfrak{N}_{\bar{w}})$ *exists and* $\tilde{\mathscr{W}}$ *satisfies the equation (2.2) for a.a.* $z\in\varrho(\mathfrak{L}_+,\mathfrak{L}_-)$.

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